BANGABASI COLLEGE

B.Sc. First year Honours Class Test-I - 2016
Subject – Mathematics
Full Marks – 50

Time – 2Hours

Answer any five questions. 5x10=50

1. (a) (i) Define an idempotent matrix. If A be an idempotent matrix of order 'n', show that $I_n - A$ is also idempotent.
   (ii) If AB = B and BA = A show that A and B are both idempotent. 3+2

   (b) If S= M+iN be a skew Hermitian matrix, then prove that
   (i) the diagonal elements of S are all imaginary or zero. 2
   (ii) M is a real skew symmetric matrix and N is a real symmetric matrix. 3

2. (a) (i) Find the angle through which the axes must be turned so that the equation lx-my+n=0 may be reduced to the form ay+b=0. 2
   (ii) Prove that the straight line lx+my+n=0 touches the circle $x^2+y^2=a^2$ if $n^2=a^2(l^2+m^2)$ 3

   or,

   Find the equation of the pair of straight lines through the origin and perpendicular to the pair of straight lines given by $2x^2+5xy+2y^2+10x+5y=0$

   (b) If (α,β) be the centroid of the triangle formed by the straight lines $ax^2+2hxy+by^2=0$ and lx+my=1 then,

   $$\frac{\alpha}{lb-mh}=\frac{\beta}{ma-lh}=\frac{2}{3(bl^2-2hlm+am^2)}$$ 5

3. Answer any two from the following: 2x5=10

   (a) Prove that the angle between two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$. Find also the angle between a diagonal and one of its adjoining sides.

(b) A variable plane has intercepts on the co-ordinate axes, the sum of whose squares is a constant k^2. Show that the locus of the foot of the perpendicular from the origin to the plane is, $(x^2 + y^2 + z^2)(x^{-2} + y^{-2} + z^{-2}) = k^2$

(c) Show that the equation of the plane through the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and perpendicular to the plane containing the lines $\frac{x}{m} = \frac{y}{n} = \frac{z}{l}$ and $\frac{x}{n} = \frac{y}{l} = \frac{z}{m}$ is, $(m-n)x + (n-l)y + (l-m)z = 0$.

4. (a) (i) $Z \neq 0$ and m is a positive integer, give an example to show that $Log\, Z^m \neq m\, Log\, Z$.

(ii) If u, v, x, y are real where $(u, v) \neq (0, \pm 1)$ and if $\tan(x + iy) = u + iv$ then show that

$u^2 + v^2 + 2u \cot 2x = 1$. 2+3=5

(b) (i) Find the least remainder when 2^{1000} is divided by 13.

(ii) Deduce from Format's theorem that every square number is of the form $5n$ or $5n \pm 1$. 2+3=5

5. (a) (i) Prove that $\sqrt[3]{5}$ is not a rational number

(ii) Let S and T be two non-empty bounded subsets of $\mathcal{R}$ and U = { x+y ; x ∈ S , y ∈ T } .

Prove that sup U = sup S + sup T

(b) Prove that every bounded infinite subset of $\mathcal{R}$ has at least one limit point . Is the condition of boundedness and infiniteness essential ? Justify your answer 5

6. (a) Let $(G,\circ)$ be a semigroup and for any two elements $a, b \in G$ each of the equations $a \circ x = b$ *and* $y \circ a = b$, has a solution in G. Prove that $(G,\circ)$ is a group.

(b) Let ABC be a triangle in which the internal and external bisectors of angle A meet the opposite side BC in D and D′; let A′ be the mid point of DD′; similarly let B′ and C′ be the mid points of EE′ and FF′ respectively which would be defined in the same fashion, show that the points A′,B′,C′ are collinear. 5

7 (a) If $I_n = \int \frac{dx}{(x^2+a^2)^n}$, n being a positive integer greater than 1 , prove that

$2(n-1)a^2\, I_n = \frac{x}{(x^2+a^2)^{n-1}} + (2n-3)I_{n-1}$ 5

(b) If $I_n = \int_0^1 x^n \tan^{-1} x\, dx$, n being a positive integer greater than 2 , prove that

$(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$ 5

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