BANGABASI COLLEGE

B.Sc. First year Honours Mid-Term Test Examination-2015

Subject – Mathematics

Full Marks – 50

Time – 2Hours

1. Answer any two questions. 2x5=10

(a) If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$ show that $A^2 - 2A + 2I = 0$. Hence find A^{50}.

(b If $(I+A)^{-1}(I-A)$ is skew symmetric and $(I+A)$ non singular matrix, prove that the matrix A is orthogonal.

(c) Prove without expanding that

$$\begin{vmatrix} 1+a_1 & 1 & 1 & 1 \\ 1 & 1+a_2 & 1 & 1 \\ 1 & 1 & 1+a_3 & 1 \\ 1 & 1 & 1 & 1+a_4 \end{vmatrix} = a_1a_2a_3a_4\left(1+\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\frac{1}{a_4}\right)$$

(d) If A is a real orthogonal matrix and $(I+A)$ is non singular, prove that the matrix $(I+A)^{-1}(I-A)$ is skew symmetric.

2. Answer any two questions: 2× 5

(a) A point moves so that the distance between the feet of the perpendiculars from it on the straight lines given by the equation $ax^2 + 2hxy + by^2 = 0$ is a constant 2d. Show that its locus is $(x^2+y^2)(h^2-ab) = d^2\{(a-b)^2+4h^2\}$.

(b) Show that the locus of the point of intersection of the tangents to the parabola $y^2 = 4ax$ at points whose ordinates are in the ratio $p^2 : q^2$ is the parabola $y^2 = (\frac{p^2}{q^2} + \frac{q^2}{p^2} + 2)ax$.

(c) Two tangents drawn to the parabola $y^2 = 4ax$ meet at an angle of 45^0. Show that the locus of their point of intersection is $(x + a)^2 = y^2 - 4ax$.

3. Answer any two questions. 2x5=10

(a) Show that the no of primes is infinite. If ϕ be the Eulers phi-function, find the value of $\phi(5040)$. 3+2

(b) State Descartes rule of sign. Apply it to ascertain the minimum number of non-real complex roots of the equation $x^7-3x^3-x+1=0$. 5

(c) If α, β, γ are three roots of $x^3-px^2+qx-r=0$, find the value of $\sum(\beta\gamma + 1/\alpha)(\gamma\alpha + 1/\beta) = ?$ 5

4. Answer any two questions. 2x5=10

(a) State completeness property of IR, the set of all real numbers. Using this property prove that a non-empty bounded below subset of IR has an infimum. 1+4

(b) State Archimedean property of IR. Show that for any positive real numbers x, there exists a positive integer m such that $m-1 \le x < m$. 1+4

(c) Define an interior point of a set in IR. When a set in IR is said to be open ? Prove or disprove that Q , the set of all rational numbers is an open set in IR. 1+1+3

(d) Define limit point of a set in IR. If $x \in IR$ is a limit point of a set S in IR then every neighbourhood of x contains infinitely many points of S.

Find the limit point(s) of the set $S = \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\ldots\ldots \frac{1}{n}, \ldots\ldots\ldots\right\}$. 1+2+2

(e) Answer any one: 5

(i) $\int \frac{dx}{3+2\sin x+\cos x}$ (ii) $\int \frac{dx}{x^4+x^2+1}$

(iii) If $\int_0^{\frac{\pi}{4}} \tan^n x\,dx$, show that $I_{n+1} - I_n = \frac{1}{n}$.

Use this relation to evaluate I_8.

5. Ans any two questions: 5x2=10

[a] Prove that any finite semigroup in which both the cancelation laws hold is a group. Is it true in case of infinite semigroup? Justify. [4+1]

[b] If f:A→B and g:B→C be both surjective, then prove that the composite mapping gof:A→C is surjective. Give an example to show that f is not surjective if gof is surjective. [3+2]

[c] Prove by vector method that $Cos(\alpha-\beta) = Cos\alpha Cos\beta + Sin\alpha Sin\beta$. [5]

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