Bangabasi College

Test Examination 2013

B. Sc. Part - I

Subject- Mathematics (Honours)

Time - 3hrs. **Paper - II** **Full Marks -100**

Module - III

(Analysis I, Evaluation of integrals)

Group - A

1. Answer any four questions : 4x10=40

a) State Completemess Axiom of IR. IF S and T be two bounded above subsets of IR, show that Sup(SUT)=Max{supS, supT}

b) State Archimedian property of IR. Using this property show that for any positive real number x, there exists a natural number m such that $m-1 \leq x < m$.

c) Prove that between two distinct real numbers there always exist a rational as well as an irrational number.

2 (a) Define limit point of a subset of IR. Prove that every bounded infinite subset of IR has at least one limit point in IR.

b) Define a closed set in IR. Prove that i) a finite set in IR is a closed set, ii) the derived set of the set $S=\left\{\frac{1}{n}; n \in \mathbb{N}\right\}$ is $\{0\}$ 5+5

3. a) Prove or disprove that no non-empty proper subset of IR is both open and closed in IR.

b) Define an enumerable set. Prove that union of two enumerable sets is enumaruble.

c) Using definition show that the set $S=\left\{\frac{1}{n}; n \in N\right\}$ is enumerable

4. a) Prove or disprove : Every bounded sequence is convergent.

b) Let {xn}n be a squence of positive real nubers such that

$\underset{n\to\alpha}{Lt}\frac{x_{n}+1}{x_n}=0$ If $0\leq\ell<1$ then prove that $\underset{n\to\alpha}{Lt}\, x_n=0$

c) Prove that the sequences $\{xn\}_n$ and $\{yn\}_n$ defined by

$x_{n+1}=\frac{1}{2}(xn+yn), \frac{2}{y_{n+1}}=\frac{1}{x_n}+\frac{1}{y_n}$ for $n\geq 1, x_1>0, y_1>0$

converge to a common limit l where $\ell^2=x_1y_1$ 2+3+5

5. a) Prove that every sequence of real nubers has a monotone subsequence.

b) State couchy's general principle of convergence. Using this principle show that the sequence $\{cn\}_n$ where $O_n= 1+\frac{1}{2}+\frac{1}{3}+........+\frac{1}{n}$ is not convergent.

c) Prove that $\underset{n\to\alpha}{Lt}\frac{1+\frac{1}{3}+\frac{1}{5}+.......+\frac{1}{2n+1}}{2n+1}=0$

6. a) Let I=(a,b) be a bounded open interval and f:I →IR be a monotone increasing function on I. Let c ←I. Then prove that

b) Let [a,b] be a bounded and closed interval and a function $f:[a,b]\to$ IR be continuons on [a,b]. Prove that f is bounded on [a,b].

$$\underset{x\to c-}{\text{Lt}}\ f(x)=\frac{\text{Sup}f(x)}{x\in(a,c)}\text{ and }\underset{x\to c+}{\text{Lt}}\ f(x)=\frac{\inf f(x)}{x\in(c,b)}$$

7. a) Prove that a function which is continuous on a closed and bounded interval is uniformly continuous there.

(b) rove that if a function $f:I\to$ IR is a Lipschitz function on I then f is uniformly continuous on I. Hence show that the function f defined by $f(x)=\log x, x\in(0,\alpha)$, is uniformly continuous on $[a,\alpha]$ where $a>0$.

Group - B

Answer any one question : 1x10

8 a) Obtain the reduction formula for the integral

$$In=\int\frac{dx}{(a+b\text{Sin}x)n}$$ where n is a positive integer greater than 1.

b) Evaluate the limit

$$\underset{n\to\alpha}{\text{Lt}}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{h^2}\right)^2.....\left(1+\frac{n^2}{n^2}\right)^n\right]^{\frac{1}{n}}$$

(5+5)

9. a) If $Im,n=\int_0^{\frac{\pi}{2}}\text{Cos}^m x\text{Sin}^n x dx$, where m,n are positive integers greater than 1, then show that

$$I_{m,n}=\frac{m-1}{m+n}\ Im\text{-}2,n\quad (m\geq 2, n\geq 2)$$

Hence show that $I_{2,2} = \frac{\pi}{16}$

b) Evaluate the Limit $\underset{n\to\infty}{Lt}\left(\frac{L^n}{n^n}\right)^{\frac{1}{n}}$ (5+5)

Module - IV
(Linear algebra, vector calculus - I)
Group - A
Answer Question number 10 and any three from the rest;

10. Answer any one :
a) Prove that

$$\begin{vmatrix} 1+a_1 & 1 & 1 & 1 \\ 1 & 1+a_2 & 1 & 1 \\ 1 & 1 & 1+a_3 & 1 \\ 1 & 1 & 1 & 1+a_4 \end{vmatrix} = a_1a_2a_3a_4(1+\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_2}+\frac{1}{a_3}+\frac{1}{a_4})$$

(5)

b) i) If A is a real orthogonal matrix and (I+A) is non-singular, prove that the matrix (I+A) -1 (I-A) is skew-symmetric. (3)
ii) A is a non-singular matrix such that the sum that the sum of the elements in each row is K, Prove that the sum of the elements in each row of A^{-1} is K^{-1}, where $K \neq O$. (2)

11. a) i) Find a real orthogonal matrix of order 3 having the elements a sthe elements $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ of a row. (3)

ii) Prove that a complex squere matrix can be uniquely expressed as the sum of a Hermition matrix and a skew Harmition matrix. (2)

b) Express the determenant $\Delta = \begin{vmatrix} S_0 & S_1 & S_2 & S_3 \\ S_1 & S_2 & S_3 & S_4 \\ S_2 & S_3 & S_4 & S_5 \\ S_1 & x & x^2 & x^3 \end{vmatrix}$ as the product of

two determenants and kence prove that $A = (x-a)(x-b)(x-c)(a-b)^2 (b-c)^2(c-a)^2$, where $Sr=a^r+b^r+c^r$ (5)

12. a) Let V be a vector space of dimension 'n' ever a field F. Prove that a linearly independent set of vectors in V is either a basis or it can be extended to a basis of V(F).

b) Reduce the follwoing quadratie form 2x2+5y2+10z2+4xy+12yz+6zx to its normal form. Find also its rank and signature.

13. a) Prove that eigen values of a real symmetric matrix are all real. (3)

b) Solve the system of equations

$$x_2+x_3=a$$
$$x_1+x_3=b$$
$$x_1+x_2=c$$

and use this solution to find the inverse of the matrix $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ (4)

c) Fidnt he dimension of the subspace S of R3 defined by
S={(x,y,z) R^3; x+2y=z, 2x+3y=y} (3)

14. a) Let A be an nxn matrix over a field F. Prove that if the eigen values of A be all distinet and belong to F, then A is diagonisable. (5)

b) Diagonalise the following matrix orthogonally

$$\begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$$

(5)

15a) Find the norm of the vector $u^2=(2,1,-1)$ is Euclidean space R^3 with respect to the usual inher product. (2)

b) Prove that in an Euclidean space, two vectors and are orthogenal iff $\| \alpha + \beta \| = \| \alpha \|^2 + \| = \| \beta \|^2$ (4)

c) Use Gram-Schmidt process to obtain an orthonoral basis of the subspace of the Euclidean space IR^3 with standard inner product, generated by the linearly independent set $\{(1,1,1),(2,-2,1),(3,1,2)\}$

Group - B

16. Answer any three questions : 3x5=15

a) Show that $\frac{|\dot{\vec{\gamma}} \times \ddot{\vec{\gamma}}|}{|\dot{\vec{\gamma}}|^3}$ is the same at all points of a curve whose vector equation is $\vec{\gamma} = (4\text{Cos}t, 4\text{S}\,\text{int}, 2t)$

b) A particle moves so that its coordinates at time t are given by $x(t)=e^{-t}\text{Cos}t$, $y(t)=e^{-t}\text{Sin}t$, $z(t)=e^{-t}$. Find the vector method its velocity and accaleration.

c) Define curl and divergence of a vector quentity. Find divergence and curl of the vector $\vec{\vartheta} = \frac{\hat{\gamma}}{r}$, where $\hat{\gamma}$ is the unit vector along and r is the magnitude of the vector $\vec{\gamma} = x\hat{i} + y\hat{j} + z\hat{k}$.

d) Prove that $\vec{\nabla}x(\phi\vec{A}) = \phi(\vec{\nabla}x\vec{A}) + (\vec{\nabla}\phi)x\vec{A}$ where is a scalar and $\vec{A}$ is a vector.

e) Define irrotational and solenoidal vectors. Show that $r^n\vec{r}$ is an irrototional vector for any value of n, but is solenoidal if n+3=0