BANGABASI COLLEGE

B.Sc. Second Year (Part-II) Honours Test Examination-2016

Subject – Mathematics

Paper - I

Full Marks – 100

Time – 4 Hours

1. Answer any two questions: 2× 4

(a) Test the convergence of the series

$(\frac{1}{2})^3 + (\frac{1.4}{2.5})^3 + (\frac{1.4.7}{2.5.8})^3 + ...$

(b) If f : [a,b]→$\mathcal{R}$ be differentiable on [a,b] , then prove that the derived function f′ cannot have jump discontinuity on [a,b].

(c) Find a and b such that $\lim_{x\to 0} \frac{x(1+a\cos 2x)+b\sin 2x}{x^3} = 1$.

2. Answer any three questions: 3× 5

(a) Prove that a convex polyhendron is a convex set. Also find the extreme points,if any of the set S = ((x,y) ; $x^2 + y^2 \geq 25$).

(b) Solve the following L.P.P. by graphical method

Minimize z = $20x_1 + 10x_2$

Subject to $x_1 + 2x_2 \leq 40$

$3x_1 + x_2 \geq 30$

$4x_1 + 3x_2 \geq 60$

$x_1, x_2 \geq 0.$

(c) Find all the basic feasible solutions of the following equations identifying in each case the basis vectors and the basic variables:

$$x_1 + 2x_2 + 3x_3 = 6$$
$$22x_1 + x_2 + 4x_3 = 4.$$

(d) Prove that every extreme point of the convex set of all feasible solutions of the system Ax = b ,x≥0 corresponds to a basic feasible solution .

(e) $x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 0$ is a feasible solution of the system of equations

$$x_1 + 2x_2 + 4x_3 + x_4 = 7$$
$$2x_1 - x_2 + 3x_3 - 2x_4 = 4$$

Find a basic feasible solution.

(f) Prove that if mixed strategies are allowed , then there always exists a value of the game problem.

3. Answer any one question: 3

(a) Find the optimal solution of the following transportation problem

(b)

	D_1	D_2	D_3	D_4	D_5	a_i
O_1	3	4	6	8	8	20
O_2	2	10	0	5	8	30
O_3	7	11	20	40	3	15
O_4	1	0	9	14	16	13
b_j	40	6	8	18	6	

(b) Solve graphically the game problem whose pay off matrix is given below

		B			
		B_1	B_2	B_3	B_4
A	A_1	2	2	3	-1
	A_2	4	3	2	6

4. Answer any TWO from the following questions:- 5X2=10

(a) Prove that if the curves $ax^2 + bx^2 = 1$ and $Ax^2 + Bx^2 = 1$ intersect at right angles, then $\frac{1}{A} - \frac{1}{a} = \frac{1}{B} - \frac{1}{b}$.

(b) Find the curvature of the curve $(x^2 + y^2)^2 = a^2(y^2 - x^2)$ at $(0, a)$.

(c) Find the equation of the cubic which has the same asymptotes as the curve

$x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$ and which passes through (0,0),(1,0) and (0,1).

5. Attempt all the question $2\times 5 = 10$

(a) A sphere of constant radius r passes through the origin O and cut the axes in A, B, C. Prove that the locus of the foot of the perpendicular from O to the plane ABC is given by $(x^2 + y^2 + z^2)^2(x^{-2} + y^{-2} + z^{-2}) = 4r^2$

(b) Show that $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$.

6. Answer any one from the following. $1\times 7 = 7$

(a) A cone of semi-vertical angle $\tan^{-1}(1/\sqrt{2})$ is enclosed in the circumscribing sphere; show that it will rest in any position.

(b) A uniform heavy elliptic lamina rests with its minor axis(2b) vertical on a rough horizontal plane. A string is attached to the centre and is pulled horizontally in the plane of the lamina, until the major axis(2a) of the lamina is vertical. Show that if there is no slipping the coefficient of friction between the lamina and the horizontal plane cannot be less than $\frac{(a^2-b^2)}{2ab}$.

7. **Answer any three questions:** 7x3=21

[a] Write the conditions so that the functional equation f(x,y) = 0 does define an implicit function. Show that the equation xySinx + Cosy = 0 determine unique implicit function in the neighbourhood of the point $\left(0,\frac{\pi}{2}\right)$. Also find the first derivative of the solution.

[b] Show that the functions u = x+y+z, v = xy+yz+zx, w = $x^3+y^3+z^3-3xyz$ are not independent but they are related by $u^3 = 3uv+w$.

[c] Let f(x,y) = $\begin{cases} x^2\sin\frac{1}{x}+y^2\sin\frac{1}{y} & when \quad x\neq 0, y\neq 0 \\ x^2\sin\frac{1}{x} & when \quad x\neq 0, y=0 \\ y^2\sin\frac{1}{y} & when \quad x=0, y\neq 0 \\ 0 & when \quad x=0, y=0 \end{cases}$

Show that $f_x(x,y)$ and $f_y(x,y)$ are discontinuous at (0,0) but that f(x,y) is differentiable at (0,0).

[d] Show that A_3, the set of even permutations of {1,2,3}is a cyclic group w.r.t product of permutations. Find a generator of this cyclic group. Answer with reason. [5+2]

[e] Prove that a group of prime order is cyclic. Give an example of a finite group which is not cyclic but whose all proper subgroups are cyclic. Justify your answer. [4+3]

8. **Answer any three questions:** 3x4

(a) Calculate the loss of K.E. in oblique impact of a smooth sphere of mass 'm' on a smooth fixed plane.

(b) A uniform chain of length '2a' is hung over a smooth peg so that the length of it on two sides are (a+b) and (a-b). If motion starts at this point of time, find the time when the chain leaves the peg.

(c) A particle of unit mass is projected with velocity 'u' at an angle 'α' with the horizon in a medium, the resistance of which is K times the velocity. Show that its direction will make an angle α/2 with the horizon after a time $\frac{1}{k}\log\left(1+\frac{ku}{g}\tan\frac{\alpha}{2}\right)$ and an angle α with the horizon after a time $\frac{1}{k}\log\left(1+\frac{2ku}{g}\sin\alpha\right)$.

(d) Obtain the velocity and acceleration of a moving particle along and perpendicular to the radius vector drawn from a fixed origin.

9. Answer any two questions: 2x3

(a) Solve the following differential equation :

$$x^2 cosy\frac{d^2y}{dx^2} + xcosy\frac{dy}{dx} - x^2 siny\left(\frac{dy}{dx}\right)^2 - siny + 1 = 0\,.$$

(b) Prove that the system of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self orthogonal where 'λ' is a parameter.

(c) Show that $y = e^{sin^{-1}x}$ is a solution of the differential equation $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$. Determine the solution of the differential equation satisfying $y(0) = 0, y'(0) = 1$.

(d) Determine the general solution of the following system of differential equations:

$$\frac{d^2x}{dt^2} - \frac{dy}{dt} = t + 1$$

$$\frac{dx}{dt} + \frac{dy}{dt} - 3x + y = 2t - 1.$$

10. Answer any two questions: 2x4

(a) Determine the eigen values and eigen functions of the boundary value problem

$$\frac{d}{dx}\left[x\frac{dy}{dx}\right] + \frac{\lambda}{x}y = 0\,, y'(1) = 0, y'(e^{2\pi}) = 0.$$

(b) Solve by Charpit's method the partial differentia l equation $px + qy = pq$ $\left(p \equiv \frac{\partial z}{\partial x},\ q \equiv \frac{\partial z}{\partial y}\right)$.

(c) Use the method of Undermined coefficients to solve the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 24e^{-3x}.$$

(d) Solve by the method of variation of parameter the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \frac{e^{-x}}{x^2}$.

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